Algebraic Pitfalls

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Adding/Subtracting Fractions:

- When adding or subtracting fractions it can be common for someone to add or subtract without having a common denominator. For example: 1/2+1/4=2/6 = 1/3. However this is incorrect.
- It may help to think of the fraction as one unit or as decimals.
- 1/2 = 0.50 and 1/4 = 0.25. If we were to add them, we have 0.75 or in fractional form 3/4 as opposed to ⅓ which about 0.33.
- If we change the denominator first we should get this. ½+¼ = 2/4+1/4 = 3/4.
- Essentially having the same denominator allows us to add like terms.

Multiplying/Division of Fractions:

- When we have a sum or difference in a fraction it is common to cancel out terms if we a common factor, however this is problematic.
- Take (1+2)/4. We can see this is 3/4. For some people they cancel the 2 and the 4 to get 1/2, which is not the same thing.
- When we think the sum or difference, we can treat that as one term.
- When we have a product in the numerator or denominator then we cancel things out, i.e. (5*6)/3 = 5(2) = 10.

Division of 3 terms vs. 4 terms

- With division of 4 terms, we have a/b/c/d and this is the same thing as a/b * d/c.
- Ex: 4 terms, 3/4/6/5 = 3/4 * 5/6 = 15/24 = 5/8.
- With division of 3 terms, there can be confusion. We have a/b/c and it is tempting to think a/b * c.
- However we should think of it as a/b/c/1 and so we have a/b * 1/c or a/bc
- Ex: 3 terms 1/2/2, we can view this as 1/2/2/1 or 1/2 *1/2 = 1/4.
- Ex: 3/2/4, we can view this as 3/2 /4/1 = 3/2 *1/4 = 3/8

Square Roots: When to Use Principal Root

- When dealing with the simplification of an expression i.e. sqrt(4), it is okay to say 2, to just have the principal root, (the positive root).
- When solving an equation you want to examine both the positive and negative solutions so that you capture all of the possible solutions.
- For example, x = sqrt(9), some would say x =3. While x = 3 is one of the solutions, it does not tell the whole story, x =3 or x =-3., in this case.

Dividing Out x and solving for x:

- When solving for x, it can be tempting to divide out an x to simplify things, but this does not tell the whole story and sometimes it may actually complicate your process.
- Example: $2x^2 + x = 0$. If you divided an x out, then you get $2x+1 = 0$. You would get $x = -1/2$. However this misses solutions.
- If you factor here, it can help. So with $2x^2 + x = 0$, it is the same as $x(2x + 1) = 0$. In order to to zero out the equation, then $x = 0$, or $2x+1 = 0$. Thus $x = 0$ or $x = -½$.
- With complicated equations, it becomes more clear that this method of dividing out x does not work. Ex: $x^2 + 5x + 6 = 0$, if you divide out a variable it gets messy quickly. We get $x + 5 + 1/x = 0$, which we don't know how to solve.

Squaring a Sum/Difference (Binomials)

- When squaring a sum or difference for a binomial, it is not uncommon to miss some terms, usually the middle term.
- Take (a+b)^2 and (a-b)^2. It is tempting to say that (a+b)^2 = a^2 + b^2 and (a-b)^2 = a^2 - b^2.
- However this misses some information and is thus not true.
- (a+b)^2 = a^2 + 2ab + b^2 and (a-b)^2 = a^2 - 2ab + b^2
- This a shortcut. If you foil things out you will find the same.
- Think about this way. (a+b)^2 = (a+b) (a+b) and (a-b)^2 = (a-b) (a-b).

Square Root of a Sum

- While you can split up roots when it comes to products, if it is a sum not so much.
- For example sqrt(xy) = sqrt(x) sqrt(y). However sqrt(x+y) does not equal sqrt(x) + sqrt (y). It does not follow the some rules.
- If it helps for the product, it makes sense. sqrt(xy) = (xy)^½. Then using exponent rules (xy)^½ = (x)^½ (y)^½ = sqrt(x) sqrt(y).
- We can’t say the same with the square root of a sum.

Improper Distribution

- It is not uncommon for some people to not distribute all the way through.
- For example 2(x-5), some might say the result is 2x-5. However they did not distribute the 2 to both terms. When you distribute a term or terms, you have to apply to the every term in the parenthesis. So you'd have 2x-10.
- It may be helpful to physically multiply things out.
- Another common distribution error is dealing with exponents.
- Some will apply the term they are distributing first.
- For example 4(x+1)^2. Some may say the result is (4x+4)^2. However exponentation comes first. We'd have to do (x+1)^2 and then multiply the result by 4.

Subtracting Polynomials

- With subtracting polynomials, it is not uncommon to miss a few things.
- The most common mistake is not applying the minus sign to the whole expression.
- One way is to apply the minus sign, treating it like you are multiplying by negative one.
- If not, you have to be extraordinarily careful.
- You could also rearrange things.

Negative Exponents

- One of the biggest misconceptions about negative exponents is that they make the numbers negative. This is very much not true.
- Negative exponents have no bearing on the sign of a number/variable.
- For example, someone may see 2^{-1} and they may say this would be equal to -2, or they may say see 3^{-2} and say it is equal $-3^2 = 9$ or -9 depending on how they read this, or $3*-2 = -6$.
- In these examples the logic is wrong.
- If we have a^{-n} where a is not zero, it just means we have $1/(a^n)$.
- For instance if we have 2^{-2}, this is equal to $1/(2^{2}) = 1/4$ as opposed to $-2^{2} = -4$

Fractional Exponents and Solving Them

- When given an equation raised to a fractional exponent it can be tempting to turn itinto radical form. Ex: $(x+1)^{2/3} = 4$
- However this does not solve anything, it makes things more complicated. Ex: cuberoot$[(x+1)^2] = 4$ or $[(x+1)^{1/3}]^2 = 4$. This is does not help us.
- Let's try getting rid of the fractional exponent.
- This can be accomplished by raising both sides by the reciprocal of the exponent.
- This in conjunction with the power of a power rule for exponents allows us to move in the direction we want to move.
- Ex: $(x+1)^{2/3} = 4$. If we take this and apply our logic then $[(x+1)^{2/3}]^{3/2} = (4)^{3/2}$. This implies that we then have $x+1 = (4)^{3/2}$.
- With $(4)^{3/2}$ we can then use radical form. We have $\sqrt{4^3}$ or $(\sqrt{4})^3$. We get 8 in either case. So we have $x+1 = 8$ or $x = 7$.